B.Sc. 6th Semester (Honours) Examination, 2023 (CBCS)

Subject : Physics

Course : CC-XIV

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group-A

1. Answer *any five* of the following questions: 2×5=10

(a) Define microstates and macrostates.

(b) How entropy is defined in statistical mechanics?

(c) Find the number of microstates of a particle of energy E, confined in a one-dimensional box of length L.

(d) In a box there are 10 cut up alphabet cards with the letters 3A's, 4M's and 3N's. We draw three cards one after another and place these on the table in the order they have been drawn. What is the probability that the word 'MAN' will appear?

(e) When an electron gas is said to be degenerate?

(f) From the statistical definition of entropy, show that the entropy of an ideal Fermi gas at $T=0$ is zero.

(g) The Fermi velocity of the electron in a metal is $0{\cdot}7\times10^{16}$ m/sec. Calculate the Fermi temperature.

(h) What do you mean by chemical potential? Consider a photon gas enclosed in a volume V and in equilibrium at temperature T. What is the chemical potential of the gas? Explain.

Group-B

2. Answer *any two* of the following questions: 5×2=10

(a) A system of two energy levels E_0 and E_1 is populated by N particles at temperature T. The particles populate the energy levels according to the classical distribution law.

(i) Derive an expression for the average energy per particle. Compute the result in the limits $T\rightarrow 0$ and $T\rightarrow\infty$.

(ii) Derive an expression for the specific heat of the system of N particles. Compute the result in the limits $T\rightarrow 0$ and $T\rightarrow\infty$. 2½+2½=5

(b) Deduce the Stefan-Boltzmann law of radiation from Planck's law.
Assume the Sun to be a black body at temperature 5800 K. Use Stefan's law to calculate the total radiant energy emitted by Sun per second. Also calculate the rate at which energy is reaching the top of the earth's atmosphere. 3+2=5

(c) Obtain an expression of the pressure (P) of an ideal Fermi gas at $T = 0$ in terms of the number density of fermions and the Fermi temperature T_f. Compare it with the pressure of an ideal classical gas at a finite temperature T. 4+1=5

(d) What is a white dwarf? What happens if a white dwarf's mass exceeds the Chandrasekhar mass? 5

Group-C

3. Answer *any two* of the following questions: 10×2=20

(a) Consider a system of an ideal Boltzmann gas of N molecules of mass m in a volume V. Show that the number of molecules in energy range $\in$ and $\in + d\in$ is given by

$$n(\in)d\in = 2\pi N\left(\frac{1}{\pi KT}\right) \in^{1/2} e^{-\in/KT}\, d\in$$

Deduce the expression for (i) the most probable energy, (ii) the number of molecules containing most probable energy, (iii) the probability at the most probable energy, and (iv) the mean energy. 4+2+2+1+1=10

(b) Consider a gas of N spinless bosons of mass m, enclosed in a volume V at a temperature T. Find an expression for each of the following:

(i) The density of single particle states as a function of single particle energy $(\in)$.

(ii) The number density $\left(n = \frac{N}{V}\right)$ of particles at a temperature T.

(iii) The Bose-Einstein transition temperature (T_C) in terms of n.

(iv) The energy density $u(T)$ and the mean occupation number $(\bar{n})$ of the ground state for $T < T_C$, in terms of T_C. 3+3+2+2=10

(c) (i) Deduce an expression for the mean occupation number of energy state ε_i assuming the particles to obey (I) F.D statistics, (II) B.E. statistics.

(ii) Derive the expression for energy density of states of an electron gas in a metal. Hence calculate the Fermi energy of a metal at 0 K. How does the Fermi energy depend on density of the electrons?

(iii) What is the average energy of electrons in a metal at 0 K? 4+5+1=10

(d) (i) The energy density of radiation at a wavelength λ is given by

$$u(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda KT}\right) - 1}.$$

Find the density distribution for two temperatures T_1 and T_2. Find an expression for the wavelength λ_{max} at which the energy density is maximum.

(ii) Starting from the expression of canonical partition function for a system having discrete energy levels, show that

$$\langle E^2 \rangle - \langle E \rangle^2 = K_B T^2 C_V$$

where C_V is the specific heat of the system. 5+5=10

———